Particle Drag and Heat Transfer in Rocket Nozzles

Donald J. Carlson*
Philco Research Laboratory, Newport Beach, Calif.

AND

Richard F. Hoglund†
Purdue University, Lafayette, Ind.

An empirical expression for the drag coefficient of a spherical particle in flow regimes such as occur in solid propellant rocket exhausts is presented. In these flows, typical particle Mach numbers will be below 2, and particle Reynolds numbers will range from less than 10^{-1} to greater than 100. Also, available heat-transfer relationships are applied to the gas-particle nozzle flow case. The effects of these relationships on computed particle velocities and temperatures are shown. In the cases considered, inertial and compressibility effects dominate for large particles and high chamber pressures, causing the thermal and velocity lags to be less than those predicted under a Stokes flow assumption. However, for small particles and low chamber pressures, rarefaction effects dominate, and the Stokes flow assumption leads to low estimates of particle lag.

It has been pointed out previously[1, 2] that the simple Stokes drag law and constant Nusselt number approximations have only limited applicability to the conditions associated with gas-particle rocket nozzle flows. The flow regimes encountered by the micron-sized particles in these flows are such that appropriate corrections must be made to account for inertial, compressibility, and rarefaction effects.

The purpose of this paper is to consolidate some of the available theoretical and experimental data for applicable drag and heat-transfer coefficients in order to provide empirical relationships useful for a wide range of rocket nozzle conditions. In addition, some results are presented to show the effect of these corrections on typical calculations of particle velocities and temperatures in rocket nozzles.

Computer calculations‡ of the flow regimes (as defined by Tsien[4, 5]) encountered by condensed-phase particles (ranging in diameter from 1 to 10 μ) in the nozzle of a small (1000-lb thrust) rocket motor operating at 400 psia are shown in Fig. 1. The Mach and Reynolds numbers are based on the relative velocities (vectorial) between the gas and the particle. The particles can be seen to follow a "corridor" through the slip and transition flow regions. The envelope of the corridor extends to Reynolds numbers greater than 100 and Mach numbers greater than 1, and thus, predictions of lag phenomena based on the Stokes drag law, restricted to continuum, incompressible flow and Reynolds numbers below about 1, may be in some error.

Available formulations for sphere-drag coefficients generally cover only a part of the flow regions encountered by particles in rocket exhaust nozzles. These formulations frequently take the form of a correction to Stokes drag law valid in either the continuum or free-molecule limit. With the motive of producing an expression for the sphere-drag coefficient, which is usable throughout the entire flow region, available expressions correcting Stokes drag law in both limits, continuum and free molecule, will be combined into a single expression. This procedure is not based upon any analytical considerations, but is merely a convenience useful for practical application.

Millikan[6] obtained a rarefaction correction to the Stokes drag law, which expresses the force F acting on a sphere of radius r immersed in a viscous fluid moving with relative velocity $v (F = 6\pi\mu r v)$. With this correction, the drag coefficient C_D is given by the formula

$$C_D = \frac{24}{Re}\left[\frac{1}{1 + (\lambda/r)(A + Be^{-Cr/\lambda})}\right] \tag{1}$$

Here, A, B, and C are constants, λ is the gas mean free path, and Re is the particle Reynolds number, given by Re =

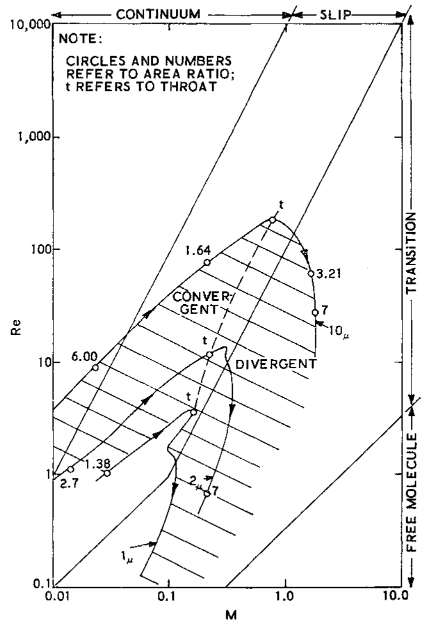

Fig. 1 Flow regimes encountered by particles in a rocket nozzle.

Received November 8, 1964; revision received May 4, 1964. This work was sponsored by the Advanced Research Projects Agency under Contract NOw 61-0905-c. The authors wish to express their appreciation to F. Marble for his comments.

* Research Scientist, Fluid Mechanics Department. Member AIAA.

† Associate Professor, School of Aeronautical and Engineering Sciences. Member AIAA.

‡ These calculations were performed using two-dimensional uncoupled gas-particle flow equations, i.e., dissipation, heat transfer, and body force effects are assumed unimportant in the calculation of the gas flow field. This program will be described in more detail elsewhere.[3]

$2\,vr\rho/\mu$ where ρ is the freestream density. Millikan included the constants B and C to account for the fact that the drag force is not linearly dependent upon Knudsen number as the flow regime varies between Stokes flow and free-molecule flow. From his oil drop experiments, Millikan obtained the values of 0.864, 0.290, and 1.25 for the constants A, B, and C, respectively. By definition, λ/r is equal to twice the Knudsen number Kn, whereas from kinetic theory, the Knudsen number can be shown to be a function of the ratio of the Mach and Reynolds numbers:

$$Kn \sim M/Re \tag{2}$$

Therefore, Eq. (1) may be re-written as

$$C_D = \frac{24}{Re}\left[\frac{1}{1 + (M/Re)(A' + B'e^{-CRe/M})}\right] \tag{3}$$

Equation (3) will be used as the basis of an empirical formulation correcting the drag coefficient for rarefaction effects. The quantities A' and B' are empirical constants determined by normalizing to free-molecular flow results of Stalder and Zurick[7]; the quantity C, representing the rate of change of drag force with Knudsen number, is left unchanged at the value of 1.25 as obtained by Millikan.§ The values of A' and B' are not kept equal to Millikan's values because first, Eq. (2) is not exact, the proportionality constants not being included, and second, the results of Stalder and Zurick and Millikan do not agree exactly, and we choose here to normalize by fitting to results of Stalder and Zurick while retaining the analytic form of Eq. (1).

The normalization in the free-molecule limit is done at $M = 0.5$ (arbitrarily selected in the center of the Mach number range of interest) where Stalder and Zurick's value of C_D for diffuse reflectance is 9.4. Assuming that the major deviation of A' and B' from A and B is due to the constants not included in Eq. (2), the ratio A'/B' is set equal to A/B. These two conditions yield $A' = 3.82$ and $B' = 1.28$, and Eq. (3) becomes

$$C_{D_r} = \frac{24}{Re}\left[\frac{1}{1 + (M/Re)(3.82 + 1.28\,e^{-1.25\,Re/M})}\right] \tag{4}$$

where the subscript r indicates that this Stokes law correction accounts for rarefaction effects.

Stokes law has been corrected for inertial effects by Torobin and Gauvin,[8] among others. Their empirical correction is

$$C_{D_i} = 24/Re[1 + 0.15\,Re^{0.687}] \tag{5}$$

Here, the subscript i indicates an inertial correction to the Stokes value. A Stokes law sphere-drag coefficient correction that accounts for both inertial and rarefaction effects may be obtained by combining the two bracketed terms on the right-hand sides of Eqs. (4) and (5). However, Fig. 1 shows that M for larger particles approaches or exceeds unity, and a correction for compressibility effects also should be included. Hoerner[9] presents a plot of the work of several previous investigators showing the effect of Mach number on the drag coefficient of spheres in the continuum flow regimes (for less than critical Reynolds numbers). An empirical fit to this curve yields

$$C_D = 0.5 + 0.5\,e^{-0.427/M^{4.63}} \tag{6}$$

The constant 0.5 in Eq. (6) is only representative of the limiting value of C_D at Reynolds numbers greater than about 1000. We will assume that compressibility effects may exist to some degree at Reynolds numbers lower than this value provided the Mach number is sufficiently large. Therefore, the value of 0.5 in Eq. (6) is replaced with the expression for C_{D_i} as expressed by Eq. (5). Furthermore, compressibility effects become negligible with increasing rarefaction regardless of Mach number, and a two-parameter exponential function of Reynolds number is postulated in order to remove the compressibility correction as mean free path increases. Equation (6) becomes

$$C_{D_{i,c}} = C_{D_i} + C_{D_i}e^{-0.427/M^{4.63}}\,e^{aRe^b} \tag{7}$$

where the subscript i,c indicates that C_D has been modified for both inertial and compressibility effects. To determine values of a and b, it is specified that the compressibility correction be 0.95 of its full value on the continuum side of the transition (at $Re = 100$) flow regime and 0.05 of its full value on the free-molecule side (at $Re = 1$); then $a = -3$ and $b = -0.88$. Equations (5) and (7) then yield

$$C_{D_{i,c}} = (24/Re)\,[1 + 0.15\,Re^{0.687}][1 + e^{-(0.427/M^{4.63}) - 3/Re^{0.88})}] \tag{8}$$

In order to include rarefaction effects, the Stokes drag coefficient, $24/Re$, in Eq. (8) is replaced with C_{D_r}. Equation (8) combined with Eq. (4) yields an expression for the sphere-drag coefficient that includes rarefaction, compressibility, and inertial effects:

$$C_D = \frac{24}{Re}\left[\frac{(1 + 0.15\,Re^{0.687})(1 + e^{(0.427/M^{4.63}) - (3.0/Re^{0.88})})}{1 + (M/Re)(3.82 + 1.28\,e^{-1.25\,Re/M})}\right] \tag{9}$$

This expression is inserted into the computer programs to compute a shifting K_D, where

$$K_D = C_D/C_{D\text{Stokes}} \tag{10}$$

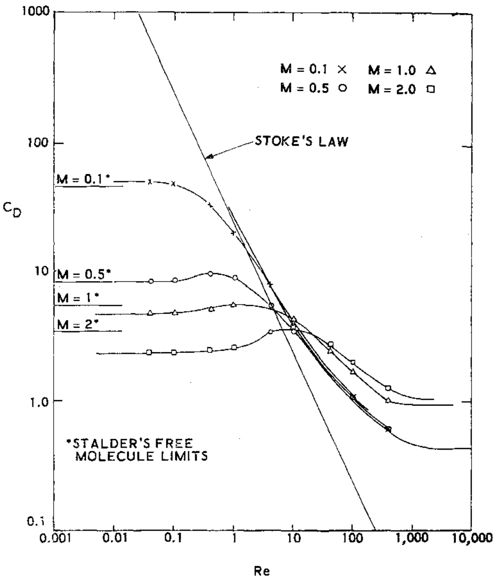

Fig. 2 Values of C_D computed from Eq. (9).

A plot of C_D, calculated from Eq. (9) vs Reynolds number, is shown in Fig. 2. The free-molecular asymptotic limits agree closely with the results of Stalder and Zurick at low Mach numbers. The deviation reaches 35% at Mach 2. The form of Eq. (9) causes the continuum values to be approached as the Reynolds number increases.

Comparison of Eq. (9) with experimental data[10–13] at $M \approx 2$ is shown in Fig. 3. These data, apart from Millikan's oil drop experiments, apparently are the lowest Mach number results available at low Reynolds numbers. Although

§ It might be more logical to use a new constant $C' = C/2.52 \times (\gamma)^{1/2}$ which would have retained the identical exponential dependence. This correction was not made, however, as the calculated values of C_D in the range of interest would be affected only slightly, the maximum difference being 9% (γ is the ratio of specific heats).

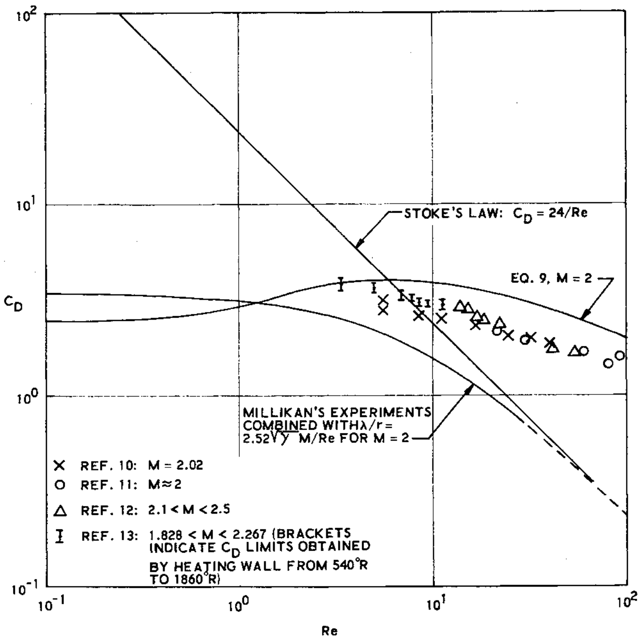

Fig. 3 C_D as a function of Reynolds number; comparison of Eq. (9) with experimental values and Stokes law.

Eq. (9) differs from the experiments by as much as 50%, the functional form of the corrections is such that a better fit cannot be obtained while retaining adequate correlation at the free-molecule limit. Better agreement would be expected at the lower Mach numbers of interest in gas-particle nozzle flows.

Millikan's formula applied at $M = 2$ by using the kinetic theory relationship between Knudsen, Mach, and Reynolds numbers is also shown in Fig. 3. It must be remembered, however, that Millikan's experiments were at very low Mach numbers, and application to $M = 2$ ignores compressibility effects. The type of data correlation presented by Sherman[14] for subsonic Mach numbers gives essentially the same result as Millikan's if it is extended to this range of Mach and Reynolds numbers. If it were available, experimental information at subsonic Mach numbers would give a better test of Eq. (9).

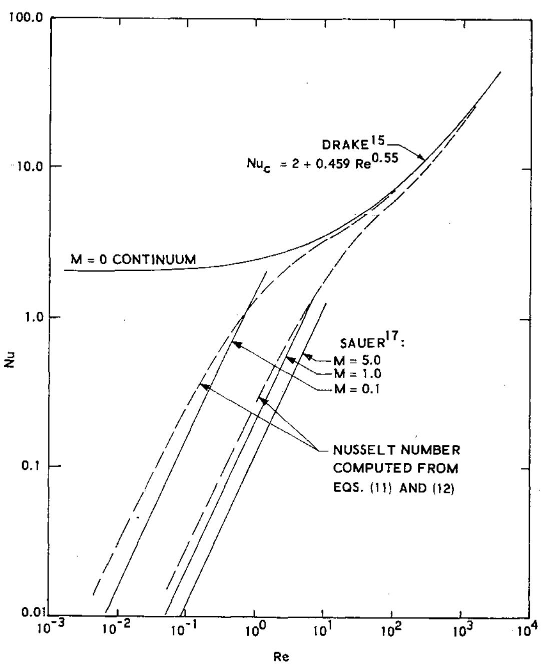

Fig. 4 Values of Nu computed from Eqs. (11) and (12).

An empirical expression for the heat-transfer correction due to inertial and rarefaction effects was obtained by combining the continuum expression used by Drake,[15]

$$Nu_c = 2 + 0.459\ Re^{0.55} Pr^{0.33} \tag{11}$$

with the transition region expression of Kavanau and Drake,[1]

$$Nu = \frac{Nu_c}{1 + 3.42(M/RePr)Nu_c} \tag{12}$$

In these equations, Nu_c is the continuum Nusselt number and

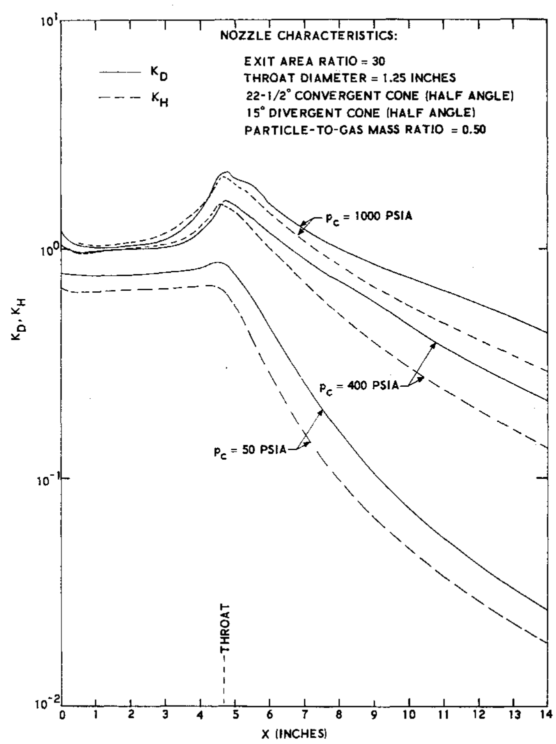

Fig. 5a K_D and K_H as a function of axial position, 2-μ particles.

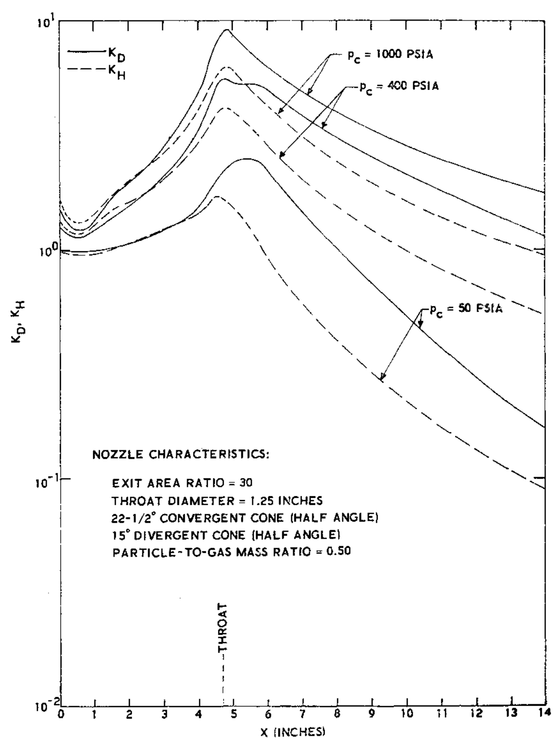

Fig. 5b K_D and K_H as a function of axial position, 10-μ particles.

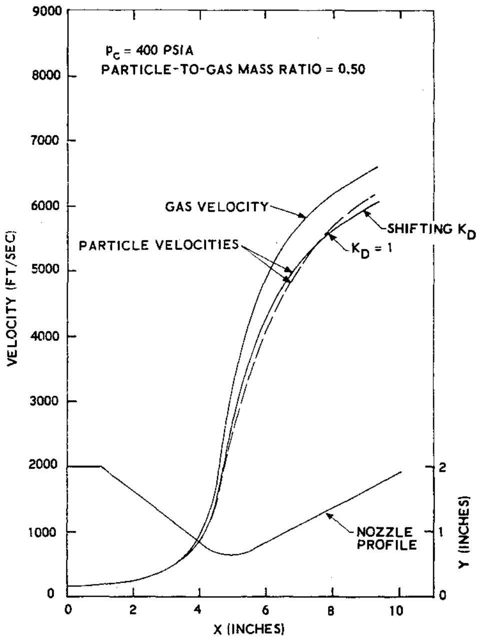

Fig. 6a Gas-particle velocity profiles, 2-μ-diam particles.

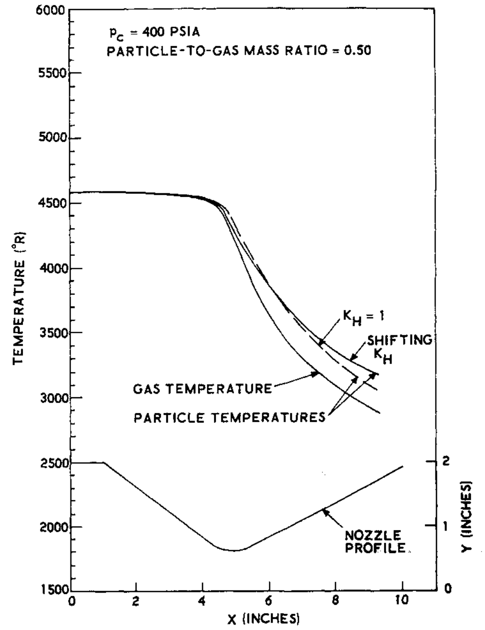

Fig. 7a Gas-particle temperature profiles, 2-μ-diam particles.

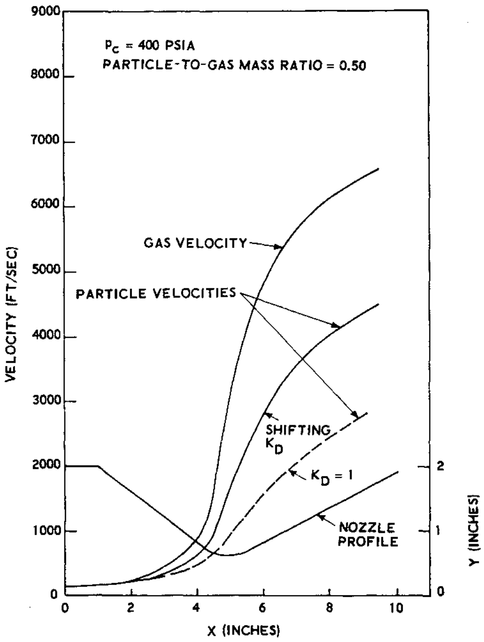

Fig. 6b Gas-particle velocity profiles, 10-μ-diam particles.

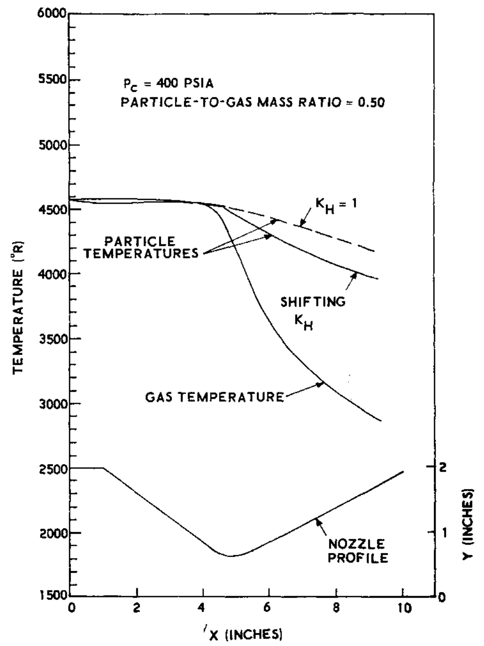

Fig. 7b Gas-particle temperature profiles, 10-μ-diam particles.

Pr is the Prandtl number. Letting $Pr = 1$, and defining $K_H = Nu/2$, one obtains

$$K_H = \frac{1}{2}\left[\frac{2 + 0.459\, Re^{0.55}}{1 + 3.42(M/Re)(2 + 0.459\, Re^{0.55})}\right] \tag{13}$$

Kavanau's correction asymptotically approaches the free-molecule values predicted by Sauer.[17] A plot of Nu vs Re is shown in Fig. 4.

Numerical calculations have been made for both K_D and K_H equal to one (Stokes flow assumption) and for K_D and K_H specified on a shifting basis by Eqs. (9–11). Typical values of K_D and K_H computed for 2- and 10-μ-diam particles flowing through a nozzle are shown for different chamber pressures p_c in Fig. 5. Early in the trajectories, when the gas density is high and particle Mach number is low, inertial effects are appreciable and K_D and K_H rise above unity for for all but the smaller particles at the low chamber pressure. As the gas density drops and lag develops, rarefaction effects become increasingly important and K_D and K_H decrease. These figures show that, for the cases considered, inertial and compressibility effects dominate for larger particles and higher chamber pressures, causing the thermal and velocity lag to be less than that predicted using the Stokes flow assumption. However, for smaller particles and lower pressures, rarefaction effects dominate and the Stokes flow assumptions lead to low estimates of the particle lag. Some actual temperature and velocity profiles are shown in Figs. 6 and 7.

Other calculations made for larger nozzles indicate that, at a given area ratio, K_D and K_H are of the same order of magnitude as for this small nozzle. That is, K_D and K_H are not strong functions of nozzle size. This is because of the importance of the Knudsen number in these flow regimes;

although velocity lags increase as nozzle size decreases, the quantity M/Re remains primarily a function of area ratio.

References

[1] Hoglund, R. F., "Recent advances in gas-particle nozzle flows," ARS J. **32,** 662–671 (1962).

[2] Gilbert, M., Allport, J., and Dunlap, R., "Dynamics of two-phase flow in rocket nozzles," ARS J. **32,** 1929–1930 (1962).

[3] Carlson, D. J., "Determination of velocity lag of condensed phases in gas-particle nozzle flows," AIAA J. (submitted for publication).

[4] Tsien, H. S., "Superaerodynamics, mechanics of rarefied gases," J. Aeronaut. Sci. **13,** 653–664 (1946).

[5] Schaaf, S. A. and Chambre, P. L., "Flow of rarefied gases," *Fundamentals of Gas Dynamics* (Princeton University Press, Princeton, N. J., 1958), Vol. III, Sec. H, pp. 687–739.

[6] Millikan, R. A., "The general law of fall of a small spherical body through a gas, and its bearing upon the nature of molecular reflection from surfaces," Phys. Rev. **22,** 1–23 (1923).

[7] Stalder, J. R. and Zurick, V. J., "Theoretical aerodynamic characteristics of bodies in a free-molecule-flow field," NACA TN 2423, p. 12 (July 1951).

[8] Torobin, L. E. and Gauvin, W. H., "Fundamental aspects of solids-gas flow," Can. J. Chem. Eng. **37,** 129–141 (August 1959).

[9] Hoerner, S. F., *Fluid-Dynamic Drag* (published by the author, 1958).

[10] Skreekanth, A., "Drag measurements in circular cylinders and spheres in the transition region at a Mach number of 2," Univ. of Toronto Institute of Aerophysics Rept. 74 (April 1961).

[11] Aroesty, J., "Sphere drag in a low density supersonic flow," Univ. of California Institute of Engineering Research Rept. HE-150-192, AD 272-244, pp. 142–143 (January 1962).

[12] Kane, E. D., "Sphere drag data at supersonic speeds and low Reynolds numbers," J. Aeronaut. Sci. **18,** 259–270 (1951).

[13] Ashkenas, H., "Low density sphere drag with equilibrium and nonequilibrium wall temperature," *Rarefied Gas Dynamics*, edited by J. A. Laurmann (Academic Press, New York, 1963), Vol. II, p. 285.

[14] Sherman, F. S., "A survey of experimental results and methods for the transition regime of rarefied gas dynamics," *Rarefied Gas Dynamics* (Academic Press, New York, 1963), Vol. II, pp. 247–248.

[15] Drake, R. M., Jr., "Discussion on G. C. Vliet and G. Leppert: 'Forced convection heat transfer from an isothermal sphere to water,' " J. Heat Transfer **83,** 170–172(1961).

[16] Kavanau, L. L. and Drake, R. M., Jr., "Heat transfer from spheres to a rarefied gas in subsonic flow," Univ. of California, Rept. HE-150-108, p. 9 (January 23, 1953).

[17] Sauer, F. M., "Convective heat transfer from spheres in a free molecule flow," J. Aeronaut. Sci. **18,** 353–354 (1951).